LOGARITHMIC COMPUTATIONS.

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It is assumed that the reader is familiar with the ordinary principles of logarithms as developed in college text-books: special attention is called to the subjects of negative characteristics, involution, evolution and the signs of the trigonometric functions in different quadrants. He is also supposed to be able to use common logarithmic tables and to get interpolated values of functions from them.

We purpose to explain the construction and use of tables containing addition and subtraction logarithms, (frequently called Gaussian logarithms or Zech), and also to describe certain habits of work by which practical computers facilitate their labor and guard against errors.

I. ADDITION AND SUBTRACTION LOGARITHMS.

If we have a given variable, $\log x$, we can make a table whose argument is $\log x$, while the corresponding function is $\log (1 + \frac{1}{x})$. Zech's addition table, (named after its computer) is such a table. In a seven-place Zech table the argument extends from 0.0000 to 6.0900, and the function from 0.3010300 to 0.0000001.

Suppose that $\log a$ and $\log b$, which is smaller than $\log a$, are given and we wish to find $\log (a + b)$. Subtracting $\log b$ from $\log a$ we got $\log \frac{a}{b}$. With this as an argument, entering the addition table we get the function $\log (1 + \frac{b}{a})$ which equals $\log (\frac{a + b}{a})$. If this function be now added to $\log a$ we evidently get $\log (a + b)$. Hence we derive the following rule:

Subtract the smaller logarithm from the larger: with the remainder as an argument take the corresponding function

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from the addition table; add this function to the larger lo-garithm.

log a	0.4263435
Zech	0.0307889
log b	9.2924389
log $\frac{a}{b}$	1.1339046
log $(a + b)$	0.4571324

In the subtraction table opposite the argument log x is found the function log $\left(\dfrac{1}{1 - \frac{1}{x}}\right)$. Hence if the argument is log $\frac{a}{b}$ the function will be log$\left(\dfrac{a}{a - b}\right)$, which subtracted from log a will give log $(a - b)$.

The process then is the same as in addition, except that the function is *subtracted* from the larger logarithm. In Zech's subtraction table when log $\frac{a}{b}$ exceeds 0.30299, it is to be sought among the functions and the corresponding argument is to be subtracted from log a. Some special cases deserve notice.

1st case. If b (the smaller) be negative and a be positive, log $(a + b)$ is found by the subtraction table, and log $(a - b)$ by the addition table; $a + b$ and $a - b$ are both positive.

2nd case. If a be negative and b positive, log $(a + b)$ is found by the subtraction table, and log $(a - b)$ by the addition table; $a + b$ and $a - b$ are both negative.

3rd case. If a and b are both negative, log $(a + b)$ is found by the addition table and log $(a - b)$ by the subtraction table; $a + b$ and $a - b$ are both negative.

4th case. If log $(b - a)$ is required, it may be easily obtained from log $(a - b)$.

II. HINTS ON SEVEN-PLACE LOGARITHMS.

Let it be required to find the mantissa of the logarithm of 2.749648. First fix the number in the mind, by repeating it thus, "twenty-seven, forty-nine " (pause) "six, forty-

eight." To be a successful and rapid computer one must be able to grasp seven figures at a glance. Find in the table the mantissa of 27496. Place the index finger of the left hand directly under the mantissa which is .4392695, and mentally find the difference between this and the next mantissa (.4392853). This difference is 158: by the aid of the table of proportional parts multiply 158 by 0.48 and add the product to the proper mantissa; obtaining in this case .4392771 which may now be written down on the computing sheet. The different operations above mentioned are to be performed mentally, nothing being written down except the final value of the mantissa; if the computer is not quick at figures the subsidiary calculations may be made on a spare sheet of paper, but never on the computing sheet. To test the accuracy of the interpolation, the tabular difference, 158, may be multiplied by 0.52 (which is $1 - 0.48$) and the product subtracted from the mantissa of 27497. In getting the logarithms of trigonometric functions the same general principles should be observed.

In using proportional parts the mind should be trained to picture for itself the partial products, one under another and then to add them.

III. ACCURACY OF INTERPOLATED FUNCTIONS.

If a seven-place logarithm be given the corresponding number is usually carried to seven significant figures; with six-place logarithms, it is carried to six figures, etc.

If an angle be obtained from the logarithm of its tangent, with seven-place logarithms, hundredths of a second are used, with six places, tenths of a second etc.: in four place computations tenths of a minute are employed, or only minutes if no great accuracy is desired.

From different logarithm tables we get the following results for A:

log tan A			
9.6438543	23°	46′	8.″51
9·643854	23	46	8.4
9.64385	23	46	·7
9.6439	23	46.3	

It is evedent that four-place logarithms do not give tenths of a minute accurately.

IV.   NEGATIVE NUMBERS.

If the logarithm of a negative number is required we get the logarithm of the corresponding positive number and write the letter n directly after it. Thus, if the number be — 2.6435, its logarithm would be written 0.42218n. The letter n does not indicate that the logarithm is negative but that the corresponding number is negative.

Skilled computers usually add 10 to all negative characteristics; it is easy from the conditions of the problem to determine whether a characteristic of 7, for instance, represents — 3 or not. In adding two logarithms whose characteristics have thus been increased, 10 is subtracted from the sum. Thus 7.234 added to 9.685 equals 6.919, 7 standing for — 3, 9 for — 1, and 6 for — 4. In subtracting 9.685 from 7.234, 10 is added to the minuend and the remainder is 7.549. Also 9.685 minus 7.234 equals 2.451. Before dividing such a logarithm by 2 (in finding the square root of a decimal number) 10 is added to it. Thus 9.685 divided by 2 equals 9.842. Before dividing by 3, 20 is added.

Number.	Logarithm.
0.064758	8.81130
$\sqrt{0.064758}$	9 40565
$\sqrt[3]{0.064758}$	9.60377
$(0.064758)^2$	7.62260
$(0.064758)^3$	6.43390

V. HINTS ON ADDITION, ETC.

In adding or subtracting two logarithms the process should be performed from left to right, the numbers being divided mentally into groups containing two or three figures each.

$$0.467489$$
$$0.248356$$
$$\overline{0.715845}$$

Thus 46 and 24 give 70, but a glance at the succeeding figures 7 and 8 shows that a unit is to be carried: hence we

write 71. 74 and 83 give 157, but again a unit is to be carried and 58 is written. The addition of 89 and 56 gives the figures 45.

An expert computer can add the two logarithms given above more rapidly than he can write the result. Since he can write more quickly from left to right than from right to left, he saves time by adding from left to right. The same order should be followed when multiplying by a small number, such as 2, 3 or 5.

VI. MISCELLANEOUS REMARKS.

Ruled computing sheets are necessary for good work. On a sheet 9 inches by 11 let a rectangle 8 by 10 be ruled and divided by other rules into 80 square inches. Let each square inch be divided by light rules into 25 equal squares. Two figures can be written in each of the smallest squares. It is well to write out a carefully arranged schedule of the work, before making the computation. Time spent in devising a convenient arrangement is by no means lost, especially when a number of computations are to be made with the same formulas.

When a constant logarithm is to be added to a number of others, it may be written near the edge of a small piece of paper and held successively over each of the others.

The computer should sit in such a position relatively to the light, that the shadow of his hand will not fall upon that part of the paper pressed by the pen.

Mental computations are frequently more accurate than those made on paper, because the attention is not distracted by the necessity of glancing from one written figure to another.

A computer ought to be very certain of the accuracy of each step of the work before proceeding to the next; otherwise his mind is troubled with doubts, and rapid, enjoyable work is impossible. It must be remembered that there can be no marked success at computing, without a severe concentration of attention and a steady purpose to work unflinchingly.

VII. LOGARITHM-BOOKS.

The following are standard works. If bought unbound,

they should be so bound that when opened they will lie open and present a flat page to the eye.

Seven place: Schron, Vega, Bruhns.

Six-place: Bremiker.

Five-place: Gauss, Gernerth (the functions of angles are given at intervals of 10"), Houel, Bremiker (the trigonometric functions are given for each hundredth of a degree)

Four-place: Bremiker (the trigonometric functions are given for each tenth of a degree), Muller, Encke, (small).

Addition and subtraction logarithms are bound up with many books: perhaps the best ones are those computed by Zech, which are carried to seven places.

If only a single book is desired, which shall be useful for all except the most refined computations, and which shall be a model in convenience of arrangement, and ease of use, Bermiker's six-place table is to be recommended. With it are bound Gaussian logarithms. An edition whose preface and introduction are in English, bears the imprint of Charles and Edwin Layton, 150, Fleet street London.

ASTRONOMICAL PHOTOGRAPHY.

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The important part that photography is likely to play in the future of astronomy renders it desirable that an opportunity should be afforded to astronomers to acquaint themselves with the improvements continually made in this branch of their science. This could best be done by the establishment at convenient places of collections designed to exhibit the progress of photography as applied to astronomical observations.

The Harvard college observatory has some special advantages for forming such a collection, since it already possesses many of the early and historically important specimens which would naturally form part of the series. Among these may be mentioned four series of daugerreotypes and photographs of various celestial objects taken in